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Polyomino Shaped Subarrays for Limited Field of View and Time Delay Control of Planar Arrays

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Introduction:

Phased array elements are often grouped into contiguous rectangular subarrays to reduce cost. This is done for two types of array applications. The first is for arrays that scan over only a limited sector, called limited field of view (LFOV) arrays. These use one phase shifter for each group of elements (subarray), and no phase shifters within the subarray. The second application is for arrays that scan over wide angles and wide frequency bands using time delay at the input to each subarray, and phase shift at every element.

Unfortunately, contiguous rectangular subarrays used in these applications have substantial quantization lobes that limit their use to systems without strict sidelobe control. Figure 1 illustrates the use of contiguous rectangular subarrays for the LFOV and time delay applications. In these examples, a microstrip patch array is divided into subarrays that are four elements wide in the ‘x’direction, and two elements wide in the ‘y’ direction. In Figure 1A, the limited field of view example, there are no phase shifters at the elements, and one phase shifter at each of the subarray input ports. This means that the subarray acts like a large element. The configuration shown achieves an 8:1 savings of phase shifters, but suffers reduced gain when the array is scanned. However, since the intention is to scan the array only through a restricted sector, then the reduced pattern gain can be tolerable.

The illustration of figure 1B shows the application of phase-shifted subarrays for a wide band application where the array is sufficiently large that the subarray peak moves off the target angle, or ‘squints’ when frequency is changed. The subarray is used here with phase shifters at every element and time-delay at the subarray input ports.

Both of these excitations break up the normal periodic phase shift within the array and develops periodic phase errors spaced one subarray apart in each plane.

In the case of a 4 x 8 subarray of half wave spaced elements, the subarray pattern is about 62º in the plane of the four elements, and 26º in the plane of the eight elements. Considering only the plane of the wider subarray dimension, this element pattern width implies that if the array is scanned in the ‘x’ direction, then the subarray pattern will have fallen off by approximately 3 dB when the array is scanned to 6.3º. Much of the rest of

the energy is in the quantization lobes that are formed because of the periodic array excitation.

Figures 2 and 3 illustrate gain reduction and the formation of quantization lobes for the two cases. In these figures u=sin θ, Figure 2 describes a linear (one dimensional) limited field of view array with eight elements in each subarray and using 10 subarrays. The curve at the upper left is the subarray pattern with its peak at broadside since there are no phase shifters at the elements. The array is excited with uniform illumination and not scanned in the curves at left. Since the array factor grating lobes are multiplied by the subarray pattern there are no quantization lobes shown in the bottom left figure. The figures at right are for the array scanned to u=0.1 (5.8 degrees). Here the subarray pattern is unchanged, but the product of the scanned array factor and the unscanned subarray pattern results in quantization lobes that are only a few dB below the main beam.

Figure 3 describes an array scanned to 30 degrees with phase shifters at each element and time delay units at the subarray input ports. The curves at left show the center frequency results, with no quantization lobes. The curves at right show the array at 1.15 fo, and here the subarray pattern peak moves off the desired scan angle to result in undesirable quantization lobes and loss in gain.

For the two dimensional array, the parameters dx and dy are the spacings between elements, and u and v are the direction cosine parameters: $u = \sin\theta\cos\phi$, and $v = \sin\theta\sin\phi$. The subarrays are N_x elements wide in the 'x' direction, and N_y elements wide in the 'y' direction.

Figures 4 and 5 illustrate the quantization lobes for planar arrays of eight element rectangular subarrays for these two applications. The subarrays are N_y elements wide in the 'y' direction ($N_y = 2$), and N_x elements wide in the 'x' direction ($N_x = 4$). Figure 4 shows the LFOV case for the array scanned to u_0=0.1, v_0=0.1 (and so θ = 8.13°.). This configuration has quantization lobes at

$$u = u_0 + p\lambda/(N_x d_x);\, v = v_0 + q\lambda/(N_y d_y); \qquad (1)$$
$$for\ p\, and\, q\ \ \pm 1, \pm 2, etc..$$

At this point the figure shows the gain reduced by several dB relative to broadside, and quantization lobes at all the relevant angles in u-v space. The largest of these is only 12 dB below the main beam.

The time delay case of Figure 5 describes an array of the same size subarrays, phase-scanned to u_0=v_0=0.5, again in a diagonal plane, and with frequency 1.15 f_0. Time delay is applied at the subarray input ports. The array elements are chosen to be a half wavelength apart at the highest frequency. Here again the quantization lobes are evident and of significant size.

Various means for suppressing these lobes have been developed, including techniques for overlapping or interleaving the subarrays [1], and each of these techniques achieves some suppression of the quantization lobes accompanied by an increase in complexity [2]. This paper is an extension of the work presented at the 2004 Antenna Applications Symposium [3] where some preliminary details described the results of using polyomino shaped subarrays and rotating the orientation of the subarrays to partially randomize the subarray phase center and patterns. This action was shown to have significantly lower sidelolbes than the peak quantization lobes of an equivalent size rectangular subarray.

The purpose of this paper is to attempt to quantify the average pattern characteristics of polyomino based subarrays in order to enable extrapolation to larger arrays. This is done for both the LFOV and time delay cases.

Choosing Polyomino Subarrays:

It is, of course, possible to break up the area of a rectangular array into as many irregular shapes as desired, and if the result is highly aperiodic, then the resulting pattern will have no quantization lobes. However it was decided in this case to select a single shape to fill an array aperture, and that shape to be composed of some number of elements p, such that p is a power of 2, so including 4,8,16 etc., because such subarrays can be fed by "lossless" orthogonal power dividers. The reason for choosing a single shape is so that a single type of power combiner will be used throughout the array (again for simplicity and reduced cost).

Polyominos (Figure 6) are shapes formed by some number 'p' of square unit cells with their sides touching, and as a domino is composed of two square unit cells, an octomino is composed of 8 square unit cells. There are a total of 5 possible tetrominos and 369 possible octominos. This investigation has considered the properties of two kinds of polyominos as subarrays, an L-tetromino and an L-octomino. These are shown in Figure 6. Many other choices could have been taken, but the selected ones are easily tiled to fit together and occupy whole arrays without leaving blank spaces that would cause unwanted sidelobes. We also avoided long thin subarrays because these cannot be fed without causing larger phase error.

The tilings shown in this study (in addition to those shown in the earlier study), were done by hand, with shapes assembled as in a puzzle. There are now computer generated tilings with both L-tetromino and L-octomino tiles, but the array size is limited in all the software we have located. Figure 7 shows the array studied throughout this paper, an array of 256 L-octominos (2048 elements) that is rotated four times and flipped so that there are eight variations of subarrays.

Average Sidelobes:

In order to obtain some measure of average sidelobe levels, we calculate the phase error variance for the groups of subarrays (and therefore for the array), and then use that data to estimate the ensemble average sidelobe levels and gain reduction. The shapes and locations of the irregular subarrays are chosen to randomize the phase errors as much as possible, but those errors are clearly not distributed according to a Gaussian probability density function. Still the following discussion uses the mathematics of random arrays, and the results of these statistical calculations will then be compared with calculations for the actual array configuration.

Irregular polyomino subarrays will, in general, have no element with its center at the configurations phase center, so one must pick an element close to the structure phase center and use it as the standard. In the LFOV case if we select one element (m_o,n_o)in a subarray to be provided the correct phase, and all the other elements in that subarray are set to have the same phase, as they would with a single feed, then the phase error at any other element (m′,n′) will be:

$$\Delta\phi = \frac{2\pi}{\lambda_0}[(m_0 - m')u_0 d_x + (n_0 - n')v_0 d_y] \qquad (2)$$

If one assumes that the subarray is rotated through four 90-degree rotations, and that the resulting phase errors when all the subarrays incorporated into the array are not correlated, then the resulting phase variance (or isotropic sidelobe level) for the subarray in the array is:

$$\bar{\varepsilon}^2 = \frac{1}{N_{Nor}}\left(\frac{2\pi}{\lambda_0}\right)^2 \sum_{m',n'}[(m_0 - m')u_0 d_x + (n_0 - n')v_0 d_y]^2 \qquad (3)$$

This expression is the phase error variance for each subarray, rotated four times, so the normalizing parameter N_{Nor} is 4 Nsub, where Nsub is the number of elements in the subarray. This variance is also the phase error variance for the whole array, again assuming that the subarray patterns or phase centers are not correlated, and so the variance above is the ensemble average isotropic sidelobe level for the array.

This expression is an ellipse when plotted against the scan angles u_o and v_o, but it is greatly simplified if the number of subarrays in the array is a multiple of 4, so that all rotations of the basic subarray are used the same number of times Further, if the location of the arbitrary phase center is kept at the same cell even when rotated, and the dimensions d_x and d_y are equal, then since $u_0^2 + v_0^2 = \sin^2\theta$, the expression as given below is a circle in u,v space.

$$\overline{\varepsilon}^2 = c\left(\frac{2\pi d_x}{\lambda_0}\right)^2 \sin^2\theta \qquad (4)$$

Similarly, for the time delay case the phase error variance (and the isotropic sidelobe level) is :

$$\overline{\varepsilon}^2 = c\left(\frac{2\pi d_x}{\lambda_0}\right)^2 (r-1)^2 \sin^2\theta \qquad (5)$$

In this equation it must be understood that λ_0 is chosen at the center frequency; (then $r = \frac{f}{f_0} = \frac{\lambda_o}{\lambda}$) and that d_x is chosen to satisfy element grating lobe conditions at the highest frequency $f_{max.}$

The constant c is evaluated from equation 3, and a similar equation for the time delay case. It is 0.5 for the L-tetromino with phase center at the position 'A', shown in Figure 6, and 1.25 for the same figure with the phase center at the position 'B'. For the L-octomino it is 0.875 for the position 'C' in Figure 6.

Figures 8 and 9 show pattern contour plots for the array of Figure 7 when using L-octomino subarrays with phase center as chosen in location 'C' of Figure 6. In this case the subarrays are used in the regular and flipped versions (although flipping the subarrays complicates the power distribution somewhat, it doesn't seem to change the peak sidelobes significantly).

Figure 10 shows a plot of equation 4 for the time delay case using the L-tetromino (phase center at 'A') and L-octomino (with phase center at 'C') configurations. The two figures have the same radial dependence, and are related simply by the ratio of c for the two cases , ie.0.5/0.875, which corresponds to about 2.4 dB.

Figure 11 plots the variance for the LFOV case for any size or type subarray, and Figure 12 plots it for the time delayed array case with the frequency ratio 'r' as parameter. In both of these figures the constant c is left out of the equation so that the results are generalized to any size or configuration with c evaluated for the particular geometry. Figure 13 plots the sidelobe level of the array of L-octomino subarrays for the time delayed subarray as a function of the frequency ratio 'r'. The plots are in dBi so they can be readily interpreted as the isotropic sidelobe level.

Gain Reduction due to Phase Error

Assuming an array element pattern that is cos θ over the upper hemisphere, and zero for the lower hemisphere, and assuming that the array factor is that of a deterministic pattern

plus uncorrelated phase errors, the gain G as a function of average isotropic sidelobe level $\bar{\sigma}^2$ (note $\bar{\sigma}^2 = \bar{\varepsilon}_0^2$), and the pattern gain without errors G_0, is

$$G = \frac{G_0}{1 + \frac{\bar{\varepsilon}^2}{4}} \qquad (6)$$

This approximate formula indicates that the fractional gain reduction is only a function of the variance (isotropic sidelobe level), and so it is the same for any size array. The actual sidelobe level relative to the main beam peak is $\bar{\sigma}^2 / G_0$, and so is continually reduced as the array becomes larger.

Peak and Average Sidelobes of Selected Distribution:

We have not attempted to apply statistical methods to estimate the peak sidelobe levels because the four rotations of the polyomino shapes still retain some symmetry that may indicate preferred angles where sidelobe peaks may exist. For this reason we have compiled results based on numerical data. The following two tables are a representative sample of that data.

<u>Results for limited field of view:</u>

	u=v=0.05		u=v=0.10		u=v=0.20	
	Sidelobe level	Gain	Sidelobe level	Gain	Sidelobe level	Gain
Rectangular subarrays						
Peak sidelobes	-19.05	35.71	-12.18	35.41	-6.25	34.26
Irregular subarrays						
Peak sidelobes	-32.7		-26.5		-21.5	
Average sidelobes						
Equation 5	-49.5	35.74	-43.1	35.55	-37.3	34.87
Integration	-46.8	35.71	-40.8	35.39	-35.9	34.15

The above table pertains to the 256 subarray aperture that is organized into rectangular (contiguous) subarrays and into the L-octomino subarrays (as shown in Figure 7). The data is given for scan angles of u=v=.05, 0.10 and 0.20. As noted, there is little gain difference between the rectangular and irregular subarrays, but there is a 14 to 15 dB reduction in peak sidelobes using the irregular subarrays. The two lines of average sidelobe values and net gain are computed to compare the results of using equation 5 with those of a direct pattern integration. The gain computation that used equation 5 began with the area gain, decreased by the scan loss (cos θ), and decreased by the taper

loss to obtain Go, which was then reduced by the phase error loss from equation 5. . This calculation agrees nearly exactly with the gain obtained by integration, and so the method will be the preferred method of calculation for larger arrays. The average sidelobes are larger for the integrated data because the integration includes all sidelobes, not just the error sidelobes. However, both sets of data indicate that the average error sidelobes are the limiting factor in determining the ultimate limits of sidelobe suppression for the irregular subarray approach. As such, equation 5 should be a good indicator of bounds as the array size is increased.

The table below shows similar data for the case of phase-scanned subarrays with time delay at the subarray input terminals. The conclusions drawn from these data are similar to those for the limited field of view data, and indicate about 13-14 dB suppression of the peak sidelobe as compared to the contiguous rectangular subarray case.

Results for time-delayed phase steered subarrays:

	r = 1.05		r = 1.10		r = 1.20	
	Sidelobe level	Gain	Sidelobe level	Gain	Sidelobe level	Gain
Rectangular subarrays						
Peak sidelobes	-25.71	34.27	-19.98	34.25	-14.47	34.16
Irregular subarrays						
Peak sidelobes	-38.68		-33.62		-28.48	
Average sidelobes						
Equation 5	-55.86	34.26	-48.21	34.22	-44.98	34.09
Integration	-53.39	34.27	-50.24	34.23	-43.1	34.1

In addition to these results, it has been shown in reference 3, that the peak sidelobe levels continue to decrease with array size, and unlike the quantization lobes of contiguous subarrays, there appear to be no sidelobes that remain a fixed ratio below the main beam. The technique therefore offers yet improved performance for larger arrays.

Conclusion:

We have presented data and theoretical conclusions to show that arrays of polyomino tile subarrays can provide enhanced sidelobe performance as compared to contiguous subarrays for the LFOV and time delayed subarray applications. We have established bounds on error variance and average sidelobe levels, and have provided data to allow extrapolation to larger arrays. We have provided some data on peak sidelobe levels, but it is felt that more study is needed to enable prediction of peak levels for larger arrays and arrays of alternative, larger polyomino subarrays.

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